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## COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE – The Earth's Magnetic Field

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AUTHOR(S) – R. H. Hilberg

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### ABSTRACT

There are in existance a number of descriptions of the earth's magnetic field in terms of expansions of spherical harmonics. A more convenient but less accurate method of calculating the geomagnetic field at a point is to use its dipole representation. This approach is developed here, along with a discussion of its accuracy, validity, and agreement with spherical harmonic expansions. A comparison of the derived dipole field with the multipolar expansion indicates that the dipole model represents the geomagnetic field near the surface of the earth to within about 10%.


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TECHNICAL MEMORANDUM

## 1.0 INTRODUCTION

The document "Natural Environment and Physical Standards for the Apollo Program" contains as a description of the geomagnetic field only an upper limit of 0.5 gauss (50 microtesla). In order to give a more complete description of the magnetic field of the earth a new model has been proposed.

There are in existance a number of descriptions of the earth's magnetic field in terms of expansions of spherical harmonics.[1][2] A more convenient but less accurate method of calculating the geomagnetic field at a point is to use its dipole representation. This approach is developed here, along with a discussion of its accuracy, validity, and agreement with the spherical harmonic expansions. A comparison of the derived dipole field with the best estimates of the geomagnetic field enables one to see the extent to which the actual field is dipolar.

## 2.0 DESCRIPTION OF THE MODEL

In this dipole field model, the geomagnetic field is represented as deriving from a simple magnetic dipole located 342 km from the center of the earth, inclined at an angle 11.5° with respect to the axis of rotation, and lying in the meridian plane with a longitude 60° West from the Greenwich Meridian.[3]

The derivation is described in more detail in the Appendix, but a brief summary is given here. The magnetic field outside the earth can be described reasonably well by the gradient of a scalar potential up to about 10 earth radii where current sheets are hypothesized.

$$B = - \text{grad } V$$

For a simple dipole field

$$V = \frac{\mu_o M \cos \theta_m}{4\pi r_m^2} = \frac{\mu_o M}{4\pi} \frac{w}{(u^2+v^2+w^2)^{3/2}}$$

where u,v,w are coordinates in the system of the magnetic dipole, with the dipole along the +w direction and the -v direction passing through the center of the earth. This dipole field is transformed to the geocentric coordinate system, $X_1$, $X_2$, $X_3$. The coordinate systems are shown in Figure 1. The transformation is given in the Appendix. The potential is then expanded in terms of spherical harmonics, and the resulting coefficients compared with the coefficients given in the literature.

When the magnetic potential is expanded in terms of the spherical harmonics in the form:

$$V = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} (g_n^m \cos m\phi + h_r^m \sin m\phi) \; P_n^m(\cos \theta)$$

the convention among geophysicists is to use Schmidt's quasi-normalized polynomials, $P_n^m(\cos \theta)$ discussed in Reference 4. These polynomials are normalized differently from the more common Legendre polynomials.

$$P_n^m(\cos \theta)_{\text{Schmidt}} = P_n^m(\cos \theta)_{\text{Legendre}} \times \frac{2^n n!(n-m)!}{(2n)!}$$

This convention will also be followed here.

## 3.0 RESULTS

The coefficients for the expansion for the geomagnetic potential predicted by the displayed, inclined dipole model are given in Table 1. The corresponding values given in the literature are given in Table 2. It is seen that the first order terms (n=1) predicted by the dipole model are in good agreement with the observed values. The second order terms agree with the observations only very roughly, and the dipole model breaks down. The second order terms amount to about 10% of the total field, so that the dipole model is accurate to about this accuracy.

The dipole model could be made more accurate by including the variation with time of the orientation and position of the magnetic dipole. As mentioned above, there is hypothesized a current sheet extending beyond 10 earth radii, so that the dipole certainly is not applicable there. In addition, the magnetosphere is distorted by the position of the sun, probably through the medium of the solar wind exerting a pressure on the magnetic cavity in the solar direction. This causes an anisotropy in the geomagnetic field beyond about 3-5 earth radii. Therefore, this model is valid to about 10% up to roughly 5 earth radii.

## 4.0 CONCLUSIONS

Comparing Table 1 with Table 2, particularly the n=2 terms indicated that the actual field at the surface of the earth differs from a dipolar field by less than 10%. If the orientation and position of the dipole had been varied to optimize agreement of the dipolar model with the observed field, it is likely the agreement would have been better.

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Attachments
- Appendix
- References
- Figure 1
- Table 1 & 2

## APPENDIX

The technique used to calculate the geomagnetic field has been to evaluate a scalar potential for a pure magnetic dipole in a coordinate system which is displaced from the dipole, and rotated with respect to the dipole axis. This coordinate system will be described generally, and then the specific parameters representative of the center of the earth relative to the effective magnetic dipole will be used to provide an estimate of the field.

The u,v,w coordinate system shown in Figure 1 is chosen such that

$$V = \frac{\mu_o M}{4\pi} \frac{w}{(u^2 + v^2 + w^2)^{3/2}} \tag{1}$$

represents the scalar potential in rationalized MKS units. The translation of the origin to the center of the earth ($x_1$, $x_2$,$x_3$ coordinate system) is described by

$$\left.\begin{aligned} x_1 &= u \\ x_2 &= v - \rho \\ x_3 &= w \end{aligned}\right\} \tag{2}$$

where $\rho$ is the distance of the dipolar axis from the center of the earth at its nearest approach. The magnetic dipole is assumed to be located at this point.

We now perform two rotations producing the geocentric coordinate system:

1. rotate around the $x_1$ axis by $\delta$
2. rotate around the $x_3$ axis by $\gamma$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (3)$$

This gives:

$$\left.\begin{aligned} u &= X_1 \cos\gamma - X_2 \sin\gamma \\ v &= X_1 \cos\gamma \cos\delta + X_2 \cos\gamma \cos\delta - X_3 \sin\delta + \rho \\ w &= X_1 \sin\gamma \sin\delta + X_2 \cos\gamma \sin\delta + X_3 \cos\delta \end{aligned}\right\} \quad (4)$$

Substituting Equation 4 into Equation 1, and expressing $X_1$, $X_2$, $X_3$, in spherical polar coordinates R, $\theta$, $\phi$, gives

$$V=\frac{u_oM}{4\pi}\,\frac{\sin\theta\cos\phi\sin\gamma\sin\delta+\sin\theta\sin\phi\cos\gamma\sin\delta+\cos\theta\cos\delta}{R^2[1+\rho^2/R^2+2\rho/R(\sin\theta\cos\phi\sin\gamma\cos\delta+\sin\theta\sin\phi\cos\gamma\cos\delta-\cos\theta\sin\delta)]^{3/2}}$$

(5)

The denominator in Equation 5 can be expanded using the binomial series:

$$(1+a)^{-3/2} = 1 - 3/2a + \frac{15}{8}a^2 + \ldots \quad a^2 < 1 \qquad (6)$$

Combining Equation 6 and Equation 5, and keeping only first order terms in $\rho/R$ ($\rho/R \sim .0537$ at the earth's surface) one obtains

$$V \doteq \frac{u_oM}{4\pi R^2}\,(\sin\theta\cos\phi\sin\gamma\sin\delta+\sin\theta\sin\phi\cos\gamma\sin\delta+\cos\theta\cos\delta)$$

$$\times \left\{ (1-3\rho/R(\sin\theta\cos\phi\sin\gamma\cos\delta+\sin\theta\sin\phi\cos\gamma\cos\delta-\cos\theta\sin\delta) \right\}$$

$$= \frac{u_o M}{4\pi R^2} \left\{ \cos\delta \; P_1^0(\cos\theta) + [(\sin\gamma\sin\delta)\cos\phi + (\cos\gamma\sin\delta)\sin\phi]P_1^1(\cos\theta) \right.$$

$$+ \frac{3\rho}{R} \; P_2^0(\cos\theta) + \frac{\rho}{R}[\sin\gamma\cos2\delta\cos\phi+(\cos\gamma\cos2\delta)\sin\phi]P_2^1(\cos\theta)$$

$$\left. + \frac{\rho}{R}[(-\tfrac{1}{4}\cos2\gamma\sin2\delta)\cos2\phi+(\tfrac{1}{4}\sin2\gamma\sin2\delta)\sin2\phi]P_2^2(\cos\theta) \right\} \quad (7)$$

For the geomagnetic field, $\rho$=342 km = .0537 earth radii; $\gamma=159^o$ , $\delta=11.5^o$, $\frac{u_o M}{4\pi}$ = 8.1 x $10^{15}$ weber-meter (or Tesla-$m^3$) = .312 gauss-(earth radii)$^3$.

Using these parameters, the values of $g_n^m$, $h_n^m$ defined by

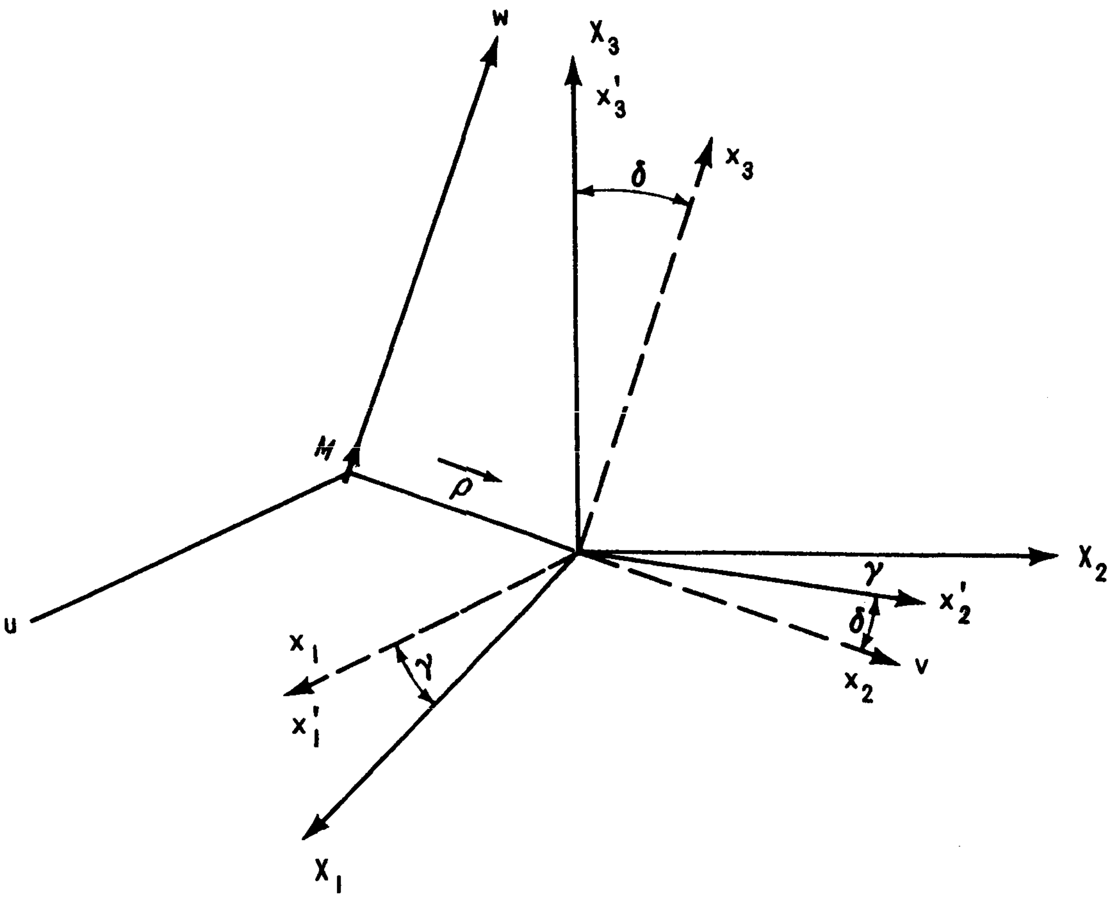
$$V = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi)P_n^m(\cos\theta)$$

are calculated for n=1 and 2 and shown in Table 1.

## REFERENCES


1. J. Cain, et al, J.G.R. 70 #15, p. 3647 (1965).
2. D. Jensen and J. Cain, J.G.R. 67, p. 3568 (1962).
3. Bartels and Chapman, "Geomagnetism", Oxford University Press, New York and London (1940).
4. A. Schmidt, NASA TT-F-8888, (1964).

FIGURE 1: COORDINATE SYSTEMS




u,v,w: MAGNETIC DIPOLE AXES; w=AXIS OF THE MAGNETIC DIPOLE $\overline{M}$, v AXIS PASSES THROUGH CENTER OF EARTH

$X_1, X_2, X_3$: GEOCENTRIC COORDINATES; $X_3$ = AXIS OF ROTATION
$X_1$ AXIS CONTAINS GREENWICH MERIDIAN

TO CONVERT FROM u,v,w:

1. TRANSLATE ALONG v A DISTANCE $\rho$, GIVING $x_1, x_2, x_3$ COORDINATES
2. ROTATE ABOUT $x_1$ BY $\delta$ GIVING $x_1', x_2', x_3'$ COORDINATES
3. ROTATE ABOUT $x_3'$ BY $\gamma$ GIVING $X_1, X_2, X_3$ COORDINATES.

## TABLE 1

Expansion Coefficients for Geomagnetic Field in gauss-(earth radii)$^3$ Calculated From Dipole Model, with Schmidt Normalization.

| | |
|---|---|
| $g_1^0 = .302$ | |
| $g_1^1 = .0224$ | $h_1^1 = -\ .0582$ |
| $g_2^0 = .0336$ | |
| $g_2^1 = -\ .0166$ | $h_2^1 = -\ .0433$ |
| $g_2^2 = .00366$ | $h_2^2 = .00495$ |

## TABLE 2

Observed Coefficients For The Geomagnetic Field in gauss-(earth radii)$^3$

| J. Cain, et.al. (Ref. 1) - 1965 | | D. Jensen and J. Cain (Ref. 2) - 1962 | |
|---|---|---|---|
| $g_1^0 = .30426$ | | $g_1^0 = .304112$ | |
| $g_1^1 = .02174$ | $h_1^1 = -.05761$ | $g_1^1 = .021474$ | $h_1^1 = -.057989$ |
| $g_2^0 = .01548$ | | $g_2^0 = .024035$ | |
| $g_2^1 = -.03000$ | $h_2^1 = .01949$ | $g_2^1 = -.051253$ | $h_2^1 = .033124$ |
| $g_2^2 = -.01574$ | $h_2^2 = -.00201$ | $g_2^2 = -.013381$ | $h_2^2 = -.001579$ |